ELLIOTT 903

Volume   2:   PROGRAMMING INFORMATION

Part   2:   PROGRAM DESCRIPTIONS

Section 13:   QATAN (B. 10)

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Chapter   1:   DESCRIPTION

1. 1   INTRODUCTION.

1. 1. 1   Purpose.

To calculate

$$t = (1/\pi) \tan^{-1}(x/y)$$

where $-1 \leq x < +1$

$-1 \leq y < +1$

1. 1. 2   Form of Distribution.

The program is distributed as a machine-code tape for input by SIR or by T2.

1. 1. 3   Method of Use.

The routine is assembled as a sub-routine of the user's program and used as a sub-routine.   It can be run at any program level and in any store module.

1. 1. 4   Accuracy.

The maximum error is $2^{-15} (\simeq .00003)$.

1. 2   FUNCTIONS.

1. 2. 1   Notation.

x, y, t are as defined in 1. 1. 1.

1. 2. 2   Number Type.

All numbers must be treated by the programmer as pure fractions.   To enable this to be done QATAN calculates

$$(1/\pi) \tan^{-1}(x/y)$$

Note, therefore, that on exit the accumulator holds the value of an angle as a fraction of π radians (180º).

1.2.3   Entry and Exit.

(for assembly by SIR.)   (for translation by T2.)

Entry: place x in QATAN+89   89;N
   "   y in QATAN+90   90;N
   "link in QATAN   0;N
   jump to QATAN+1   1;N

(where N is the number of
the block)

Exit: the result is in the accumulator

x and y are not affected.

1.2.4   Identifiers.

QATAN must be declared as a global identifier in all blocks of a SIR program which refer to it.

On the library tape a mnemonic label and identifier list are separated from the coding by several inches of blank tape: the mnemonics must not be loaded into the tape reader if the tape is to be translated by T2.

1.3   ERROR INDICATIONS.

If x=y=0 blank tape is output continuously.

1.4   METHOD USED.

a)   The program computes

$$a = \begin{cases} |x/y| & \text{if} \quad |x/y| < 1 \\ |y/x| & \text{if} \quad |x/y| \ge 1 \end{cases}$$

and applies the transformation

$$Z = \frac{a - (\sqrt{2} - 1)}{(3 - 2\sqrt{2})\,a + (\sqrt{2} - 1)}$$

Note $|Z| < 1$

b) $\qquad S = 1/\pi \, \tan (\sqrt{2} - 1) Z$

is calculated by a Chebyshev series.

c) $\qquad$ The final result is found by forming

$$u = (1/\pi) \, \tan^{-1} |x/y| = \begin{cases} \frac{1}{8} + S \text{ if } |x/y| < 1 \\ \frac{1}{2} - (\frac{1}{8} + S) \text{ if } |x/y| \geq 1 \end{cases}$$

and the value found according to the table below:

	$y \geq 0$	$y < 0$
$x/y \geq 0$	u	u - 1
$x/y < 0$	-u	1 - u

1.5   TIME TAKEN.

The time taken depends on the values of x/y and of y.

The maximum time is about 3. 2 milliseconds.

1.6   STORE USED.

128 consecutive store locations and the appropriate B-register.